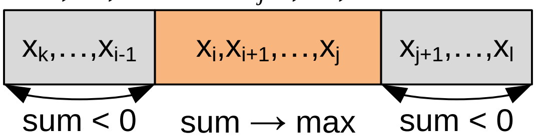
Sequences

E-OLYMP **2585. Profit** The cows have opened a new business, and Farmer John wants to see how well they are doing. The business has been running for n ($1 \le n \le 100000$) days, and every day i the cows recorded their net profit P_i ($-1000 \le P_i \le 1000$).

Farmer John wants to find the largest total profit that the cows have made during any consecutive time period. (Note that a consecutive time period can range in length from one day through n days.) Help him by writing a program to calculate the largest sum of consecutive profits.

► In the problem we need to find a subsequence of consecutive numbers that will have the maximum possible sum among all possible such subsequences. If the maximum sum is attained on subsequence $x_i, x_{i+1}, \ldots, x_j$, then for any k, $1 \le k < i$ and l, $j < l \le n$, the sum of elements $x_k, \ldots, x_{i-1}$ and $x_{j+1}, \ldots, x_l$ will be negative.



Kadane's algorithm. Move through the array from left to right and accumulate the current partial sum in the variable s. If at some moment s turns out to be negative, then we assign $s = 0$. The maximum of all values of the variable s during the passage through the array will be the answer to the problem.

Consider a sequence X given below. Construct the partial sums. The current value of the partial sum is set to zero when the current sum becomes less than zero and we start counting the sum from the next number. The maximum value among all partial sums is 6, which is the answer. The required subsequence is 4, -2, 4.

X	5	-3	1	-7	4	-2	4	-1	-8	2
s	5	2	3	-4 0	4	2	6	5	-3 0	2
				s = 0					s = 0	

E-OLYMP **6198. Minimum sum** Two arrays of positive integers are given $a_{1..n}$ and $b_{1..n}$. Find the permutation $i_1, i_2, \ldots, i_n$ of numbers $1, 2, \ldots, n$, for which the sum

$$a_1 * b_{i1} + \ldots + a_n * b_{in}$$

is minimal. Each number must be included in permutation only once.

► Sort array a in ascending order and array b in descending order. Then the required minimum sum is

$$\sum_{i=1}^{n} a_i \cdot b_i$$

Consider how to get the maximum sum for the input sample.

a_i	2	3	4	7	10	
b_i	11	9	6	6	5	
$a_i * b_i$	22	27	24	42	50	165

E-OLYMP **5621. Find a multiple** Given n positive integers, each is not greater than 15000. This numbers are not necessarily different (so it may happen that two or more of them will be equal). Your task is to choose a *few* of given numbers ($1 \le few \le n$) so that the sum of chosen numbers is multiple for n (i.e. $n * k$ = (sum of chosen numbers) for some integer k).

► Let $d_1, d_2, \ldots, d_n$ be the input numbers. Consider all partial sums $s_i = d_1 + \ldots + d_i$. Since we have exactly n partial sums, then among all values of s_i mod n there must be either two identical or such i that s_i mod $n = 0$.

If s_{a-1} mod $n = s_b$ mod n for $a - 1 < b$, then $d_a + \ldots + d_b$ is divisible by n. Set of numbers $d_a, d_{a+1}, \ldots, d_b$ will be the answer. If there is such i that s_i mod $n = 0$, then the answer is $d_1, d_2, \ldots, d_i$.

Consider the sample given. Compute all partial sums:

s_i	s_i mod 5
$s_1 = 1$	1
$s_2 = 1 + 2 = 3$	3
$s_3 = 1 + 2 + 3 = 6$	1
$s_4 = 1 + 2 + 3 + 4 = 10$	0
$s_5 = 1 + 2 + 3 + 4 + 1= 11$	1

i	1	2	3	4	5
d_i	1	2	3	4	1
s_i	1	3	6	10	11
s_i *mod 5*	1	3	1	0	1

There are several required sets. For example:

- since $s_1 = s_3$, then $d_2 + d_3 = 5$ is divisible by 5.
- since $s_1 = s_5$, then $d_2 + d_3 + d_4 + d_5 = 10$ is divisible by 5.
- since $s_3 = s_5$, then $d_4 + d_5 = 5$ is divisible by 5.
- since $s_4 = 0$, then $d_1 + d_2 + d_3 + d_4 = 10$ is divisible by 5.